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Comments on D-brane dynamics near NS5-branes

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Abstract: We study the properties of a D-brane in the presence of k NS5 branes. The Dirac-Born-Infeld action describing the dynamics of this D-brane is very similar to that of a non-BPS D-brane in ten dimensions. As the D-brane approaches the fivebranes, its equation of state approaches that of a pressureless fluid. In non-BPS D-brane case this is considered as an evidence for the decay of the D-brane into "tachyon matter". We show that in our case similar behavior is the consequence of the motion of the D-brane. In particular in the rest frame of the moving D-brane the equation of state is that of a usual D-brane, for which the pressure is equal to the energy density. We also compute the total cross-section for the decay of the D-brane into closed string modes and show that the emitted energy has a power like divergence for $D0$, $D1$ and $D2$ branes, while converges for higher dimensional D-branes. We also speculate on the possibility that the infalling D-brane describes a decaying defect in six dimensional Little String Theory.

Keywords: D-branes, Tachyon Condensation.

Contents

1. Introduction

Recent years were marked by considerable progress in the understanding of time-dependent physics in string theory. One of the most studied subjects in this direction is the D-brane dynamics in the presence of an unstable (tachyonic) mode in the worldvolume theory. The condensation of this mode leads to a new final time-independent state in the theory. For example in the case of non-BPS D-branes the condensation of the tachyonic mode may lead to a more stable brane configurations or to complete annihilation of the brane. These systems can be studied by perturbing the boundary CFT corresponding to the D-brane by exactly marginal operator corresponding to the tachyon [1]–[11]. The recent development also showed that many properties of unstable D-branes are surprisingly well captured by Dirac-Born-Infeld (DBI) type actions [12]–[17], which turns out to be very useful tool for studying the D-brane decay. In particular the DBI action analysis shows that the stress-energy tensor of the final state of the decaying D-brane is that of pressureless fluid. This fact is considered as evidence for the decay of an unstable D-brane into pressureless fluid of heavy closed string modes–"tachyon matter". This observation lead to the proposal of tree level open-closed string duality [18, 19], which states that the open string theory, *without an explicit coupling to the closed strings* contains all the necessary information about the closed string modes that the D-brane decays into.

Another example of a D-brane with an unstable mode in the worldvolume theory was recently considered in the paper by D. Kutasov [20] (see also [21, 22] for work along these lines). The setup discussed in this paper involved an (asymptotically) BPS Dp-brane, which is placed at some distance from a stack of k parallel NS5-branes in type-II string theory. This configuration is not supersymmetric and hence unstable–the D-brane will experience an attractive force, which will move it towards the NS5 branes. This process is closely related to the tachyon condensation problem. Indeed, as we will briefly review below, the DBI actions describing the radial motion in the NS5 brane system and the

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tachyon decay for non-BPS branes are related for large values of the tachyon field.[1] Hence this system may serve as a useful model for studying the decay of non-BPS D-branes in ten dimensions. In particular the geometric interpretation of the tachyon direction may open new venues for understanding this problem better.

In this paper we make further progress in understanding the properties of a D-brane in the presence of NS5 branes. In section 2 we briefly review results of [20] and show that the apparent decay of the D-brane into pressureless fluid, described by DBI action, is due to the fact that the D-brane moves with the speed close to the speed of light. It turns out that in the rest frame of the D-brane the stress-energy tensor of it is that of an ordinary D-brane with pressure equal to the energy density. Hence we conclude that the DBI action describes only the classical open string motion and the radiative corrections due to closed string emission should be accounted for separately.

In section 3 we compute the closed string emission and show that the total energy of the closed strings emitted is divergent for Dp-branes with $p \leq 2$, while finite for $p > 2$. This seems to suggest that the classical open string theory analysis is valid only for Dp-branes with $p > 2$, while for lower dimensional branes the large radiative corrections render the classical trajectory invalid.

In section 4 we speculate on the possibility that the infalling D-brane describes holographically a decaying defect in the six dimensional Little String Theory (LST) [24]. We would like to argue that this defect is described by the same DBI action as the infalling D-brane and the amplitude of the emission of little strings from the defect is [25]

$$I(\epsilon) = i \int dt \rho(t) e^{i\epsilon t} , \tag{1.1}$$

where $\rho(t)$ is related to the stress-energy tensor (pressure) as follows

$$T_{ij} \sim \rho(t) \delta_{ij} . \tag{1.2}$$

Then we show that the exponential part of full (non-perturbative) density of states of LST at high energies is such that it exactly cancels the exponentially divergent part of the total cross-section and hence this process is very reminiscent of the non-BPS D-brane decay.

In section 5 we discuss our results and mention some directions for future work.

2. Stress-energy tensor from Lorentz transformation

In this section we show that the motion of D-branes described in [20] using the DBI action does not include the backreaction from the radiation of closed strings. In particular we show that the stress-energy tensor, calculated in [20] is that of a usual D-brane boosted to the speed close to the speed of light.

We start by reviewing the results of [20] on the radial motion of a Dp-brane in the background of k NS5 branes. Let x^μ, $\mu = 0, 1, \ldots, 6$ be the coordinates along the worldvolume of $NS5$-branes, while x^n, $n = 6, 7, 8, 9$ are labeling the transverse directions. Then the

[1] It can be shown [23] that the DBI actions are identical for all values of tachyon field if one considers a slightly more complicated system where one of the directions transverse to the NS5-branes is compactified.

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background fields around k $NS5$-branes are given by the CHS solution [26]. The metric, string coupling (dilaton) and NS-NS B field are

$$\begin{aligned} ds^2 &= dx^\mu dx_\mu + H(x^n) dx^m dx^m \equiv g_{MN} dx^M dx^N \\ \frac{g_s(\Phi)}{g_s} &= e^{(\Phi-\Phi_0)} = \sqrt{H(x^n)} \\ H_{mnp} &= -\epsilon_{qmnp} \partial^q \Phi \,, \end{aligned} \tag{2.1}$$

where $H(x^n)$ is the harmonic function describing k NS5 branes, g_s is the asymptotic string coupling and H_{mnp} is the field strength of the B field. We will be interested in the case of coincident fivebranes in which case $H(x^n)$ reduces to

$$H(r) = 1 + \frac{k l_s^2}{r^2} \,, \tag{2.2}$$

with $r^2 = x^n x_n$ and $l_s = \sqrt{\alpha'}$.

Let us study the radial motion of a D-brane stretched in $(x^1, \ldots x^p)$ in this background. Without loss of generality we can label the worldvolume of the D-brane by $(x^0, x^1, \ldots, x^p)$. The position of the D-brane in the radial direction gives rise to a scalar field $r(x^\mu)$ in the worldvolume theory. The dynamics of this field is described by DBI action

$$S_p = -\tau_p \int d^{p+1}x e^{-(\Phi-\Phi_0)} \sqrt{-det(G_{\mu\nu} + B_{\mu\nu})} \,, \tag{2.3}$$

where τ_p is the asymptotic tension of the Dp-brane

$$\tau_p \sim \frac{1}{g_s l_s^{p+1}} \,, \tag{2.4}$$

$G_{\mu\nu}$ and $B_{\mu\nu}$ are the induced metric and the B field respectively

$$\begin{aligned} G_{\mu\nu} &= \frac{\partial x^M}{\partial x^\mu} \frac{\partial x^N}{\partial x^\nu} g_{MN} \\ B_{\mu\nu} &= \frac{\partial x^M}{\partial x^\mu} \frac{\partial x^N}{\partial x^\nu} B_{MN} = 0 \,. \end{aligned} \tag{2.5}$$

In the last line we used the fact that the only non-zero components of the B field are in the angular, transverse directions.

In this paper we will discuss the spatially homogeneous motion of the D-brane. Then r is a function of $x^0 = t$ only and using (2.3) we find the following action describing the dynamics of this field

$$S_p = -\tau_p V \int dt \sqrt{\frac{1}{H(r)} - \dot{r}^2} \,, \tag{2.6}$$

We will be mainly interested in the dynamics of the D-brane deep in the CHS throat, that is for $r \ll \sqrt{k} l_s$. We would also like to take the asymptotic g_s to zero keeping the rescaled radial coordinate $R \equiv g_s^{-1} r$ fixed. This limit is usually taken in the holographic description of the Little String Theory [27]. In this regime the 1 in the harmonic function $H(r)$ can be neglected and the action takes the following form

$$S_p = -T_p V \int dt \frac{R}{\sqrt{k} l_s} \sqrt{1 - \left(\frac{d}{dt} \log R\right)^2}\,, \tag{2.7}$$

where we introduced the rescaled D-brane tension $T_p \equiv \tau_p g_s$. The action (2.7) can be further simplified by introducing

$$e^{\frac{\phi}{\sqrt{k} l_s}} \equiv \frac{R}{\sqrt{k} l_s}\,. \tag{2.8}$$

In the new variables we find

$$S = -T_p V \int dt e^{\frac{\phi}{\sqrt{k} l_s}} \sqrt{1 - \dot{\phi}^2}\,. \tag{2.9}$$

This action is very reminiscent of the DBI action for the tachyon of the non-BPS D-brane (see e.g. [16])

$$S \sim \int dt V(T) \sqrt{1 - \dot{T}^2}\,, \tag{2.10}$$

where

$$V(T) = \frac{1}{\cosh \frac{\alpha T}{2 l_s}}\,. \tag{2.11}$$

Here α is 1 for bosonic string and $\sqrt{2}$ for superstring. We see that for large T the map $\phi \to -T$, maps (2.9) into (2.10). Moreover for $k = 2$ the action (2.9) is mimicking superstring, while for $k = 4$ the bosonic string.

The equation of motion following from the action (2.9) is

$$1 = -\frac{\ddot{\phi} \sqrt{k} l_s}{1 - \dot{\phi}^2}\,. \tag{2.12}$$

It can be easily solved, yielding

$$e^{-\frac{\phi}{\sqrt{k} l_s}} = A \cosh \frac{t}{\sqrt{k} l_s}\,. \tag{2.13}$$

It is useful to express the constant A in terms of the conserved energy of the brane

$$E = \dot{\phi} \frac{\partial L}{\partial \dot{\phi}} - L = T_p V \frac{e^{\frac{\phi}{\sqrt{k} l_s}}}{\sqrt{1 - \dot{\phi}^2}}\,. \tag{2.14}$$

Substituting this expression into (2.13) we find

$$e^{-\frac{\phi}{\sqrt{k} l_s}} = \frac{T_p V}{E} \cosh \frac{t}{\sqrt{k} l_s}\,. \tag{2.15}$$

In order to find the stress-energy tensor we should consider the full DBI action, involving spatial excitations along the D-brane

$$S_p = -T_p \int d^p x dt e^{\frac{\phi}{\sqrt{k} l_s}} \sqrt{1 - \dot{\phi}^2 + (\partial_i \phi)^2} \equiv \int \mathcal{L} dt d^p x\,. \tag{2.16}$$

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Then using the definition

$$T_{\mu\nu} = \phi_{,\mu}\frac{\partial\mathcal{L}}{\partial\phi_{,\nu}} + \mathcal{L}\eta_{\mu\nu}\,, \tag{2.17}$$

we find[2]

$$\begin{aligned} T_{00} &= T_p\frac{e^{\frac{\phi}{\sqrt{k}l_s}}}{\sqrt{1-\dot\phi^2}} = \frac{E}{V} \\ T_{ij} &= -T_p e^{\frac{\phi}{\sqrt{k}l_s}}\sqrt{1-\dot\phi^2}\,\delta_{ij}\,. \end{aligned} \tag{2.18}$$

The equations of motion imply that the T_{00} is conserved, while the pressure is exponentially decaying

$$T_{ij}V = -\frac{E}{\cosh^2\frac{t}{\sqrt{k}l_s}}\,. \tag{2.19}$$

A similar behavior of the pressure in the related problem of tachyon decay is usually considered as evidence for the decay of an unstable D-brane into a pressureless fluid of "tachyon matter" [18, 19]. We show below that here this behavior can be actually explained from the fact that the D-brane is moving with the speed close to the speed of light. Indeed consider the rest frame of the infalling D-brane at $t = t_0$

$$\begin{aligned} \tilde\phi &= \frac{\phi-\phi(t_0)-v(t_0)(t-t_0)}{\sqrt{1-v^2(t_0)}} \\ \tilde t &= \frac{t-t_0-v(t_0)(\phi-\phi(t_0))}{\sqrt{1-v^2(t_0)}}\,, \end{aligned} \tag{2.20}$$

where $v(t_0) = \dot\phi(t_0)$. One can show that in this frame (at $t = t_0$) the only non-zero components of stress energy tensor take the form

$$\begin{aligned} \tilde T_{00} &= M\delta(\tilde\phi) \\ \tilde T_{ij} &= -M\delta_{ij}\delta(\tilde\phi)\,, \end{aligned} \tag{2.21}$$

where $M = T_p e^{\frac{\phi(t_0)}{\sqrt{k}l_s}}$. Indeed, transforming (2.21) to the (ϕ, t) frame, we find the following non-zero components of stress-energy tensor

$$\begin{aligned} T_{00}(t_0) &= \left(\frac{\partial\tilde t}{\partial t}\right)^2\tilde T_{00} = \frac{E}{V}\delta(\phi-\phi(t_0)) \\ T_{ij}(t_0) &= \tilde T_{00} = -\frac{E}{V}\frac{1}{\cosh^2\frac{t_0}{\sqrt{k}l_s}}\delta(\phi-\phi(t_0)) \\ T_{0\phi}(t_0) &= \frac{\partial\tilde t}{\partial t}\frac{\partial\tilde t}{\partial\phi}\tilde T_{00} = -\frac{v(t_0)}{\sqrt{1-v^2(t_0)}}T_p e^{\frac{\phi(t_0)}{\sqrt{k}l_s}}\delta(\phi-\phi(t_0))\,. \end{aligned} \tag{2.22}$$

[2]It should be understood that the stress-energy is localized at the position of the D-brane, i.e. it involves delta functions in the directions transverse to the D-brane, and in particular should have a factor of $\delta(\phi-\phi(t))$. The $T_{\mu\nu}$ computed from the DBI action can be thought of as stress-energy tensor integrated over transverse directions.

The first two lines can be readily compared to (2.18), while the third line is the energy density flow (momentum) in the ϕ direction, and can also be obtained from the DBI action[3]

$$T_{0\phi} = -\frac{\delta \mathcal{L}}{\delta \dot{\phi}}\,. \tag{2.23}$$

3. Emission rate from the one point function

In this section we use worldsheet conformal field theory results to compute the emission of closed strings from the infalling D-brane. Our main interest here is to establish whether the radiative corrections spoil the classical result obtained from the DBI analysis, i.e. whether the total emitted energy is divergent or not. Hence we will be somewhat cavalier about the overall constants.

The string theory deep in the CHS throat (2.1) is described by an exact conformal field theory

$$CFT = \mathbb{R}^{5,1} \times \mathrm{SU}(2)_k \times \mathbb{R}_\phi\,. \tag{3.1}$$

The first factor in (3.1) describes the space-time directions along the fivebranes. The second factor describes the angular three-sphere.[4] The radius of the three-sphere is[5]

$$R_{\text{sphere}} = \sqrt{k}\,. \tag{3.2}$$

Finally, the third factor in (3.1) describes the linear dilaton direction ϕ

$$\Phi = -\frac{Q}{2}\phi\,; \qquad Q = \frac{2}{\sqrt{k}}\,. \tag{3.3}$$

The number of fivebranes k determines the level of the SU(2) current algebra and the slope of the dilaton in (3.1). More precisely, since (3.1) is a background for the superstring, the worldsheet theory contains fermions; the total level k of the SU(2) current algebra receives a contribution of $k-2$ from the bosons, and $+2$ from the fermions. The total central charge is

$$6 + \frac{3(k-2)}{k} + \left(1 + \frac{3}{2}Q^2\right) + 10\cdot\frac{1}{2} = 6 + \frac{3(k-2)}{k} + \left(1+\frac{6}{k}\right) + 10\cdot\frac{1}{2} = 15\,, \tag{3.4}$$

which is the correct value for the superstring.

The bosonic part of a normalizable closed string vertex operator in this background can be schematically represented as

$$V_b = P(\text{oscillators})\Phi_{j,m,\bar{m}} e^{Q\tilde{j}\phi} e^{i\vec{k}\cdot\vec{X}} e^{i\epsilon X^0}\,, \tag{3.5}$$

where $\Phi_{j,m,\bar{m}}$ is a primary of $\mathrm{SU}(2)_{k-2}$ WZW, $\vec{k}$ is the momentum in $\mathbb{R}^5$, ϵ is the energy and

$$\tilde{j} = -\frac{1}{2} + i\lambda\,, \qquad \lambda \in \mathbb{R}_+\,, \tag{3.6}$$

[3]The third line directly follows from energy conservation $\partial^\nu T_{0\nu} = 0$.

[4]As is well known, CFT on a three-sphere with a suitable NS $B_{\mu\nu}$ field is described by the SU(2) WZW model.

[5]In the rest of the paper we set $l_s = 1$ for simplicity.

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labels normalizable operators in the linear dilaton theory. P(oscillators) is a polynomial in oscillators for SU(2) WZW, $\mathbb{R}^{5,1}$ and $\mathbb{R}_\phi$. We will be interested in NS-NS and RR sector closed string vertex operators, which can be obtained by coupling V_b to fermions in the standard way.

The infalling D-brane is described by a boundary state $|B\rangle$ in this conformal field theory and serves as a time-dependent source for closed string modes. The emission of closed strings is described in terms of the one point function. More precisely the amplitude for the emission of a closed string mode V is [25]

$$\mathcal{A} \sim \frac{U(V)}{\sqrt{\epsilon}} = \frac{\langle V|B\rangle}{\sqrt{\epsilon}}\,, \tag{3.7}$$

where ϵ is the energy of the emitted mode. The boundary state $|B\rangle$ can be naturally factorized into

$$|B\rangle = |B\rangle_{\mathbb{R}^5} \times |B\rangle_{\mathrm{SU}(2)} \times |B\rangle_{t,\phi}\,. \tag{3.8}$$

Let us discuss each of the factors in turn. The first factor in $|B\rangle$ is especially simple and can be schematically presented as

$$\int d^{5-p}k \sum_{\psi\in\mathcal{H}_L} e^{i\phi(\psi)}|\psi\rangle_L|\psi\rangle_R|k\rangle\,, \tag{3.9}$$

where the sum goes over all left-right symmetric oscillators states, which are unit normalized. The phase $e^{i\phi(\psi)}$ can in principle be determined, but since we will be interested in the absolute value square of the amplitude, this phase is irrelevant for us.

The second factor is the boundary state in $\mathrm{SU}(2)_k$ WZW model. In general there are $k+1$ different boundary states corresponding to BPS D-branes [28, 29] (along with the $\bar{D}$-branes, which have an opposite sign in front of RR component)

$$|\tilde{l}\rangle = \frac{1}{\sqrt{2}}\sum_{2j=0}^{k}\left(\frac{S_j^l}{\sqrt{S_j^0}}|j\rangle\rangle_{NS} + \frac{S_j'^l}{\sqrt{S_j'^0}}|j\rangle\rangle_R\right), \tag{3.10}$$

where l and j are half-integers labeling the SU(2) primaries,[6] and S_j^l, $S_j'^l$ are the components of the modular transformation matrix. $|j\rangle\rangle$ is the Ishibashi state [30]

$$|j\rangle\rangle_{NS,R} = \sum_{\psi\in\mathcal{H}_{NS,R}} e^{i\phi(j,\psi)}|j,\psi\rangle_L|j,\psi\rangle_R\,, \tag{3.11}$$

where the sum goes over all left-right symmetric, unit normalized states constructed over current algebra primary $|j\rangle$. As in the flat space case, the phase $e^{i\phi(j,\psi)}$ is not relevant for our purposes. We will also see that the particular form of S_j^l affects only the overall normalization of the total cross-section, thus we do not need to write down it explicitly. The D-brane that we are interested in, is the one that looks like a point on S^3, corresponds to the $\tilde{l}=0$ boundary state.

[6]The primary $|j\rangle$ has SU(2) spin j.

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Finally let us discuss the last factor in (3.8). This boundary state describes the motion in the (ϕ, t) plane. As discussed above the trajectory of the infalling D-brane is given by

$$e^{-\frac{\phi}{\sqrt{k}}} = A \cosh \frac{t}{\sqrt{k}} \,. \tag{3.12}$$

Performing Wick rotation $t \to iY$ we find

$$e^{-\frac{\phi}{\sqrt{k}}} = A \cos \frac{Y}{\sqrt{k}} \,. \tag{3.13}$$

This is the supersymmetric version of the bosonic hairpin brane of [31]

$$\frac{1}{2} \tilde{r} \exp\left(-\frac{X}{\sqrt{k}}\right) - \cos \frac{Y}{\sqrt{k+2}} = 0 \,, \tag{3.14}$$

and was recently discussed in [32]. The one point function of closed string vertex in the presence of a hairpin brane is

$$U^{\nu}(P, Q) \sim \frac{\Gamma(-i\sqrt{k}P)\Gamma(1 - i\frac{P}{\sqrt{k}})}{\Gamma(\frac{1}{2} + \nu + \frac{\sqrt{k}}{2}(Q - iP))\Gamma(\frac{1}{2} - \nu - \frac{\sqrt{k}}{2}(Q + iP))} \,, \tag{3.15}$$

where P and Q are momenta in the ϕ and Y directions and $\nu = 0, 1/2$ for NS-NS and RR sectors respectively. The one point function in the RR sector is obtained from that of the NS-NS sector by 1/2-spectral flow.

We would like to analytically continue the (3.15) to find the closed string emission amplitude for the infalling D-brane. The naive analytical continuation $Q \to i\epsilon$, $\nu \to i\nu$ gives

$$U^{\nu}(P, \epsilon) \sim \frac{\Gamma(-i\sqrt{k}P)\Gamma(1 - i\frac{P}{\sqrt{k}})}{\Gamma(\frac{1}{2} + i\nu + i\frac{\sqrt{k}}{2}(\epsilon - P))\Gamma(\frac{1}{2} - i\nu - i\frac{\sqrt{k}}{2}(\epsilon + P))} \,. \tag{3.16}$$

Taking absolute value square, we find the cross-section for emission of a string mode

$$|U^{\nu}|^2 \sim \frac{1}{\sinh \pi P\sqrt{k} \sinh \frac{\pi P}{\sqrt{k}}} (\cosh(\pi\epsilon\sqrt{k} + \nu) + \cosh \pi P\sqrt{k}) \,. \tag{3.17}$$

We see that the leading contribution comes from small P, and it diverges exponentially with the energy

$$|U|^2 \sim e^{\pi\sqrt{k}\epsilon} \,. \tag{3.18}$$

This answer is physically unacceptable. The reason for which the naive analytic continuation does not work is explained in [32] and is essentially due to the fact that the Y coordinate of the hairpin is effectively compact, while the ϵ is unbounded.

On the other hand the one point function can be directly computed (at least semiclassically) by integrating the wavefunction of the closed string mode over the profile of the brane. In the NS-NS sector we find

$$U(P, \epsilon) = \int d\rho d\tau e^{2\rho} e^{i\tilde{\epsilon}\tau} e^{(-1+is)\rho} \delta\left(e^{\rho} \cosh \tau - \frac{1}{2}\tilde{r}\right) , \tag{3.19}$$

where

$$s = -\sqrt{k}P\,; \qquad \tau = \frac{t}{\sqrt{k}}\,; \qquad \tilde{\epsilon} = \epsilon\sqrt{k}\,; \qquad \rho = \frac{\phi}{\sqrt{k}}\,, \tag{3.20}$$

and the factor of $e^{2\rho}$ comes from the measure in the ϕ direction. Note that the δ-function used in the calculation differs from the one obtained from (3.12) by a τ dependent factor. This normalization is natural, since as we will see below, in this case the one point function will depend on $\tilde{r}$ only through a phase. The physical quantities such as the absolute value square of the one point function should not depend on $\tilde{r}$, since it can be shifted away by ϕ redefinition. The integral can be brought to the form

$$U = \left(\frac{\tilde{r}}{2}\right)^{is} \int_{-\infty}^{\infty} d\tau e^{i\tilde{\epsilon}\tau} (\cosh\tau)^{-1-is}\,. \tag{3.21}$$

Before performing the integral note that the main contribution to it comes from $|t| < \sqrt{k}$ region, which means that we are in the weak coupling region (at least if the initial energy of the brane E is large enough), hence our calculation is reliable. Performing the integral we find the semiclassical expression for the one point function

$$U = (\tilde{r})^{is} \frac{\Gamma(\frac{1}{2} + \frac{i\sqrt{k}}{2}(\epsilon - P))\Gamma(\frac{1}{2} - \frac{i\sqrt{k}}{2}(\epsilon + P))}{\Gamma(1 - iP\sqrt{k})}\,. \tag{3.22}$$

The exact answer is [32]

$$U = (\tilde{r})^{is} \frac{\Gamma(\frac{1}{2} + \frac{i\sqrt{k}}{2}(\epsilon - P))\Gamma(\frac{1}{2} - \frac{i\sqrt{k}}{2}(\epsilon + P))}{\Gamma(1 - iP\sqrt{k})} \Gamma\left(1 - \frac{iP}{\sqrt{k}}\right). \tag{3.23}$$

The one point function in the RR-sector can be easily obtained by 1/2-spectral flow, which amounts to a shift in energy $\epsilon \to \epsilon + 1/\sqrt{k}$. We will see that only the high energy behavior of the amplitude is relevant for our purposes, which is the same for NS-NS and RR sectors, hence in the rest of this section we will restrict the discussion to the NS-NS sector.

Now we can write down the full cross-section for the emission of a closed string, which has momentum P in the linear dilaton direction, $\vec{k}_\perp$ in the directions in $\mathbb{R}^5$ transverse to the D-brane, labeled by $j \times j$ primary in S^3 and has total oscillator level n

$$|U(j, P, \epsilon(k_\perp, j, P, n))|^2 \simeq \frac{\pi^2}{k} \frac{\sinh(\pi P\sqrt{k})}{(\cosh\pi\sqrt{k}\epsilon + \cosh\pi\sqrt{k}P)\sinh\frac{\pi P}{\sqrt{k}}} S_0^j\,. \tag{3.24}$$

The energy is determined from the physical state condition[7]

$$\epsilon = \sqrt{P^2 + k_\perp^2 + 4n + \frac{j(j+1)}{k}}\,. \tag{3.25}$$

Let us compute the total average number and the energy of particles produced during the decay. As mentioned above we would like to see whether either of these quantities blows up. From the form of the cross-section for the emission we conclude that the only divergence

[7]We neglected $\mathcal{O}(1)$ constant since at any rate we are interested in the high energy behavior.

can come from the high energy/large P region. In this region we can neglect the term $j(j+1)/k$ in ϵ and then the only j dependence of the cross-section comes from the factor S_0^j. Moreover we need to know only the asymptotic density of closed strings that the D-brane can decay into. It is clear that the D-brane can emit only left-right symmetric closed string modes, since the boundary state $|B\rangle$ describing the D-brane is left-right symmetric.

The asymptotic density of left-right symmetric states is[8] [33]

$$d_n \sim n^{-\frac{q}{2}-\frac{1}{2}} e^{\pi\sqrt{4n}\sqrt{\frac{2k-1}{k}}}, \tag{3.26}$$

where $2q = 6$ is the number of non-compact spatial directions. The average number of particles can be approximated by

$$\begin{aligned} N \sim \sum_j S_0^j \int dP dl d^d k_\perp l^{-q} \frac{1}{2\sqrt{P^2+l^2+k_\perp^2}} \times \\ \times \exp\left(-\pi\sqrt{k}\sqrt{P^2+l^2+k_\perp^2} + \pi\sqrt{k}P - \frac{\pi}{\sqrt{k}}P + \pi l\sqrt{\frac{2k-1}{k}}\right), \end{aligned} \tag{3.27}$$

where $l^2 = 4n$, d is the number of directions in the $\mathbb{R}^5$ transverse to the brane and we neglected some overall k-dependent factors in the integral. It is convenient to express the integral in spherical coordinates

$$\begin{aligned} N \sim \int d\theta d\phi dr (\cos\theta)^{d-1} (\sin\theta)^{1-q} (\sin\phi)^{-q} r^{d-q} \times \\ \times \exp\left(-\pi r\left(1 - \sin\theta\left(\cos\phi\left(1-\frac{1}{k}\right) + \sqrt{\frac{2k-1}{k^2}}\sin\phi\right)\right)\right), \end{aligned} \tag{3.28}$$

where

$$\begin{aligned} r &= \sqrt{k}\sqrt{P^2+l^2+k_\perp^2} = \sqrt{k}\epsilon\,, \\ P\sqrt{k} &= r\sin\theta\cos\phi\,, \\ l\sqrt{k} &= r\sin\theta\sin\phi\,, \\ k_\perp\sqrt{k} &= r\cos\theta\,. \end{aligned} \tag{3.29}$$

Introducing

$$\begin{aligned} \cos\chi &= 1 - \frac{1}{k}\,, \\ \sin\chi &= \sqrt{\frac{2k-1}{k^2}}\,, \end{aligned} \tag{3.30}$$

we see that the expression in the exponent of (3.28) is manifestly non-positive

$$\exp\left(-\pi r\left(1-\sin\theta\left(\cos\phi\left(1-\frac{1}{k}\right)+\sqrt{\frac{2k-1}{k^2}}\sin\phi\right)\right)\right) = \exp(-\pi r(1-\sin\theta\cos(\phi-\chi)))\,, \tag{3.31}$$

and hence the integral can only have power-like divergences in r, which come from the region $\phi \sim \chi$ and $\theta \sim \pi/2$.

[8] Authors of [32] estimated the total cross-section using the full density of states, which seems to be inconsistent with the left-right symmetry of the boundary state.

Performing the integral by steepest descent, we find

$$N \sim \int dr r^{\frac{d}{2}-q-\frac{1}{2}} . \tag{3.32}$$

Similarly one can find the total emitted energy

$$\mathcal{E} \sim \int dr r^{\frac{d}{2}-q+\frac{1}{2}} . \tag{3.33}$$

We see that the emitted energy is finite for Dp-branes with $p > 2$ and infinite for $p \leq 2$. This result is identical to the result of similar calculation in the case of rolling tachyon [25]. This seems to suggest that the tree level open string theory analysis is valid only for Dp-branes with $p > 2$, while for lower dimensional branes the large radiative corrections render the classical trajectory invalid. It is still plausible that the decay process of the lower dimensional branes is dominated by particles with energies (masses) comparable to the energy of the original brane.[9] In this case the transverse momenta of the emitted particle with the energy ϵ are

$$\begin{aligned} \langle k_\perp \rangle &= \epsilon \sqrt{\frac{2}{\pi r}} \langle \tilde{\theta} \rangle \sim \sqrt{\frac{2\epsilon}{\pi \sqrt{k}}} \\ \langle P \rangle &\sim \epsilon (1 - \frac{1}{k}) . \end{aligned} \tag{3.34}$$

Hence we conclude that from the ten dimensional point of view the emitted particles are ultrarelativistic (at least for large k), that is they are moving with the speed close to the speed of light in the radial direction. On the other hand, from the point of view of a six dimensional observer living on the fivebranes, these particles are highly non-relativistic. We see that the six dimensional properties of the decay products in our case are again very reminiscent of the ten dimensional properties of the emitted particles in the non-BPS D-brane decay.

4. Holographic description

In the previous sections we saw that there is a striking similarity between the decay of non-BPS D-brane in ten dimensions and the properties of (asymptotically) BPS D-brane infalling onto a stack of NS5 branes. In particular we saw that from the point of view of six-dimensional observer living on the fivebranes the decay products have exactly the same properties as the decay products of a non-BPS D-brane in ten dimensions. This leads us to the following proposal: *The infalling BPS D-brane describes holographically a defect in the six dimensional Little String Theory, which resembles a non-BPS D-brane in type-II string theory.*

In this section we attempt to describe this defect directly in the LST and thus provide further evidence for the proposal. To do that we will need to postulate few properties of

[9] Here we are referring to the $6d$ masses of the particles, as measured by the observer living on fivebranes. The $10d$ mass of the particles emitted can actually be quite small (for large k); from $10d$ point of view the main contribution to the energy is coming from the momentum in the linear dilaton direction.

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defects in the LST, which make them very similar to non-BPS D-branes in ten dimensions. Using these properties we will reproduce some of the results of section 3.

1. The low energy behavior of a defect extended in $p+1$ space-time directions is described by the DBI action (2.16), in which the ϕ field is not regarded as a geometric direction but rather as a tachyon.

2. Furthermore we require, following [25], that amplitude of the little string emission by the defect in LST be given by

$$I(\epsilon) = i \int dt \rho(t) e^{i\epsilon t} \,, \tag{4.1}$$

where $\rho(t)$ is the (rescaled) pressure computed from the DBI action (2.19)

$$\rho(t) = \frac{1}{\cosh^2 \frac{t}{\sqrt{k}}} \,. \tag{4.2}$$

3. The LST has a Hagedorn density of states [34] with the temperature

$$T_H = \frac{1}{2\pi\sqrt{k}} \,. \tag{4.3}$$

Not all possible LST states can be emitted in the decay of a defect, but rather “left-right” symmetric states,[10] i.e. only the square root of the total Hagedorn spectrum will enter into the calculation of the total cross-section.[11]

Let us now compute, using these assumptions, the total emission of little strings from the decaying defect. First we should compute the Fourier transform of the pressure

$$\int \frac{e^{i\epsilon t}}{\cosh^2 \frac{t}{\sqrt{k}}} dt = 2\sqrt{k} \int \frac{e^{\tau} e^{\frac{i\epsilon\sqrt{k}}{2}\tau}}{(1+e^{\tau})^2} d\tau \,. \tag{4.4}$$

The integral can be computed by closing the contour at infinity and picking up the contributions from the poles

$$I(\epsilon) \sim \frac{\epsilon}{\sinh \frac{\pi\epsilon\sqrt{k}}{2}} \sim \epsilon e^{\frac{-\pi\epsilon\sqrt{k}}{2}} \,. \tag{4.5}$$

Hence we see that the leading behavior of the cross-section for producing a state at energy ϵ is

$$|I(\epsilon)|^2 \sim \epsilon^2 e^{-\pi\epsilon\sqrt{k}} \,. \tag{4.6}$$

[10]One might wonder why, while in section 3 only the perturbative density of states of ten dimensional theory entered the calculation, here we are insisting on using full Hagedorn spectrum of LST, which has many more states. This is a standard situation in holographically related theories, e.g. in LST in order to compute the high energy density of states we need to know only the perturbative spectrum in the background of NS5 branes.

[11]We should emphasize that this last assumption is very speculative, since unlike in perturbative string theory, we do not have any reason to believe that the states in Little String Theory can be factorized into the product of left and right movers.

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Then the total energy emitted is

$$\mathcal{E} \sim \int dM d^{5-p} k_{\perp} |I(\epsilon)|^2 \sqrt{\rho_H(M)}\,, \tag{4.7}$$

where $\rho_H(M)$ is the full Hagedorn density of states of LST

$$\rho_H(M) \sim e^{2\pi\sqrt{k}M}\,, \tag{4.8}$$

and the integration goes over the mass of the little string modes M and the momenta $k_{\perp}$ transverse to the defect. The square root of the Hagedorn density of states gets canceled by the exponential part of $|I(\epsilon)|^2$, and one should look at the next to the leading behavior of $\rho_H(M)$ to actually compute the emitted energy. For the second quantized Little String Theory this was computed in [35], but since the LST is strongly interacting theory this result is not directly applicable here. We conclude that, just as in section 3, the emitted energy has power like behavior, although we were not able to compute this power. This result suggests that our assumptions about the properties of defects in LST outlined above are correct and the defects in six dimensional LST behave in the same way as non-BPS D-branes in ten dimensions.

5. Discussion

In this note we studied the properties of an (asymptotic) BPS D-brane in type-II string theory in the presence of k parallel NS5 branes. The classical open string dynamics of the D-brane is described by DBI action. As D-brane approaches to the NS5 branes its equation of state approaches to that of a pressureless fluid. We showed that the apparent decay of the D-brane into pressureless fluid is due to its motion. More precisely we found the equation of state of the D-brane in its rest frame and it is that of a usual brane with the pressure equal to the energy density. This indicates that the DBI action does not describe the closed string emission and the radiative corrections should be accounted for separately. Next we computed the closed string emission, using exact CFT results, and found that the emitted energy is finite for Dp-branes with $p > 2$, while has power divergence for the lower dimensional branes. Hence we concluded that the classical open string theory analysis is not applicable for the $D0$, $D1$ and $D2$ branes. This result is very reminiscent of the conclusion that one arrives to in the case of the decay of non-BPS Dp-branes [25], namely the emitted energy is finite for $p > 2$ and diverges for $p \leq 2$. This lead us to the following proposal: *The infalling BPS D-brane describes holographically a defect in the six dimensional Little String Theory, which resembles a non-BPS D-brane in type-II string theory.* Next we attempted to describe the defect directly in Little String Theory to provide further evidence for our proposal. Under assumptions, which are very natural from the non-BPS D-brane prospective, we reproduced some of the exact CFT results. Namely we showed that the emitted energy in LST has power like behavior, thus confirming that the defects in six dimensional LST behave in the same way as non-BPS D-branes in ten dimensions.

In conclusion we would like to discuss possible directions for future work. It would be interesting to generalize the results of this paper to other holographically related theories. For example one could consider the dynamics of a $D1$ brane in the near horizon geometry of N $D3$ branes. In this case the two holographically related theories are the type-IIB theory on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ supersymmetric Yang Mills theory in four dimensions [36]. A $D1$-brane will be attracted to the $D3$-branes [37] and it is plausible that, just as in the system considered in this paper, it will describe a decaying one dimensional defect in $\mathcal{N} = 4$ SYM. It would be interesting to identify this object and check whether the descriptions on two sides agree.

A more ambitious program would be to describe holographically non-BPS D-branes in ten dimensions as some kind of moving defects in a higher dimensional theory, thus making the tachyon direction geometric. At the moment it is not clear whether such a description is possible.

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